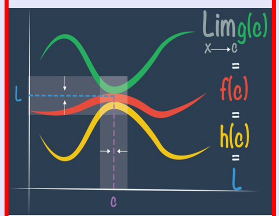
Calculus I

Lecture 30



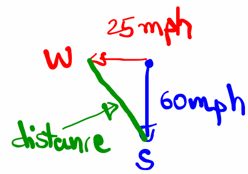
Feb 19-8:47 AM

Two cars moving from the same point.

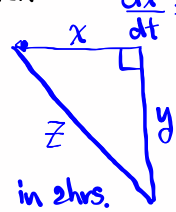
one goes South @ 60 mph.

other one goes west @ 25 mph.

At what rate is the distance between them changing 2 hrs later?



$\frac{dx}{dt} = 25\,mph$



$\frac{dy}{dt} = 60\,mph$

$\frac{dz}{dt}$ in 2hrs.

$z^2 = x^2 + y^2$

$2z\frac{dz}{dt} = 2x\frac{dx}{dt} + 2y\frac{dy}{dt}$

$\frac{dz}{dt} = ?$

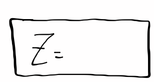
in 2 hrs

$x = 2 \cdot 25 = 50$

$y = 2 \cdot 60 = 120$

$z^2 = x^2 + y^2$

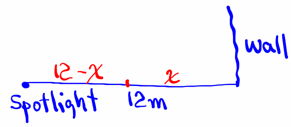
$z^2 = 50^2 + 120^2$

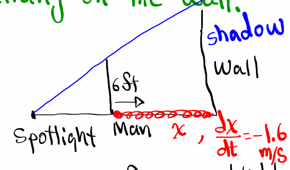


we can find $\boxed{\frac{dz}{dt}}$

Apr 8-8:47 AM

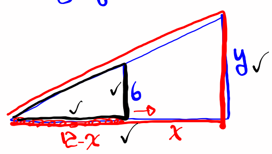
A spotlight is on the ground 12 m away from a wall. It is shining on the wall.





A man is 2m tall walks away from spotlight toward the wall @ 1.6 m/s.

How fast the length of his shadow on the wall changing when he is 4 m from the wall?



$$\frac{12-x}{6} = \frac{12}{y}$$

$$(12-x)y = 12 \cdot 6$$

$$12y - xy = 72$$

$x = 4$

$$\frac{12-4}{6} = \frac{12}{y}$$

$\boxed{y =}$

$$12\frac{dy}{dt} - \left[\frac{dx}{dt} \cdot y + x \cdot \frac{dy}{dt}\right] = 0$$

Solve for $\boxed{\frac{dy}{dt}}$

Apr 8-8:54 AM

$f(x) = 3x^4 - 16x^3 + 18x^2$ $\boxed{-1 \le x \le 4}$

$f(-1) = 3(-1)^4 - 16(-1)^3 + 18(-1)^2$

$= 3 + 16 + 18$

$\boxed{f(-1) = 37}$

$f(4) = 3(4)^4 - 16(4)^3 + 18(4)^2$

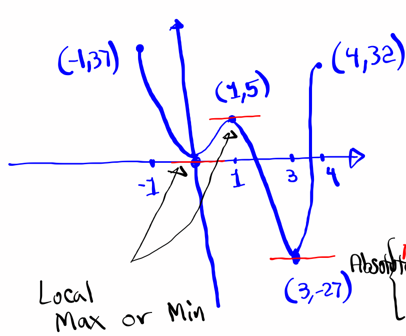
$= 768 - 1024 + 288$

$\boxed{f(4) = 32}$

$f'(x) = 12x^3 - 48x^2 + 36x$

$f'(x) = 0 \rightarrow 12x(x^2 - 4x + 3) = 0 \rightarrow 12x(x-1)(x-3) = 0$

$x = 0 \quad x = 1 \quad x = 3$



we have horizontal tan. lines at $x = 0$, $x = 1$, $x = 3$

$f(0) = 0$, $f(1) = 5$, $f(3) = -27$

Absolute Max. Point is 37 at $x = -1$

Absolute Min. Point is -27 at $x = 3$

Apr 8-9:08 AM

Extreme Value theorem

If $f(x)$ is a continuous function on $[a,b]$, then $f(x)$ has abs. max and abs. min for some values in $[a,b]$.

Fermat's theorem:

If $f(x)$ has local max or min at some value c, and if $f'(c)$ exists, then $f'(c)=0$.

Critical number:

It is $\boxed{\text{a number}}$ in the domain of $f(x)$ (c)

such that $f'(x)=0$ or $f'(x)$ is undefined.

$f'(c)=0$ $\quad$ $f'(c)$ is undefined

Apr 8-9:19 AM

$f(x)= \underbrace{x^2-4x-12}_{\text{cont. } f(x)}$, $[0,5]$

$f(0)=-12$

$f(5)=5^2-4(5)-12$

$=25-20-12$

$=-7$

$f'(x)=2x-4$

$f'(x)=0 \rightarrow x=2$

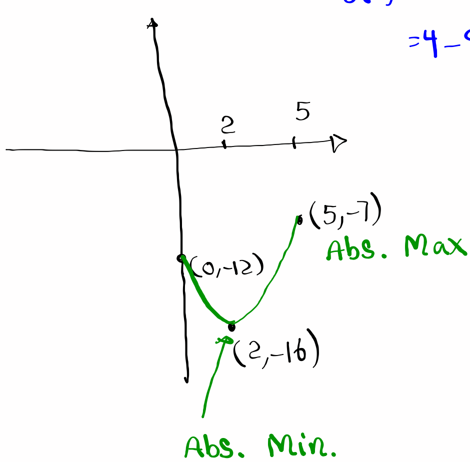
Critical number

$f(2)=2^2-4(2)-12$

$=4-8-12=-16$

$(2,-16)$

Critical Point



Apr 8-9:26 AM

$f(x) = \underbrace{(x^2-1)^3}$, $[-1, 2]$

Polynomial

cont. everywhere

$f(-1) = 0$

$f(2) = 27$

$f'(x) = 3(x^2-1)^2 \cdot 2x$

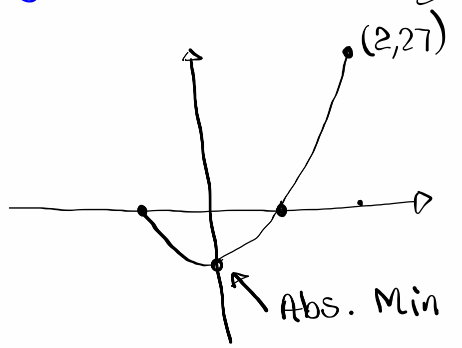
$f'(x) = 6x(x^2-1)^2$

$f'(x) = 0$ $\quad 6x(x^2-1)^2 = 0$

$x = 0$ $\quad x = \pm 1$

C.N.

Abs. Max
(2,27)



C.P.

$(0, -1)$

$(-1, 0)$ $\quad (1, 0)$

Abs. Min $(0, -1)$

Apr 8-9:31 AM

$f(x) = \dfrac{x}{x^2 - x + 1}$ $\quad [0, 3]$

$f(0) = 0$

$f(3) = \dfrac{3}{3^2 - 3 + 1} = \dfrac{3}{7}$

$f'(x) = \dfrac{1(x^2-x+1) - x(2x-1)}{(x^2-x+1)^2}$

$f'(x) = \dfrac{x^2 \cancel{-x} + 1 - 2x^2 \cancel{+x}}{(x^2-x+1)^2}$

$f'(x) = \dfrac{1 - x^2}{(x^2-x+1)^2}$

$f'(x) = 0 \to 1 - x^2 = 0 \to x = \pm 1$

$f'(x)$ undefined $\to x^2 - x + 1 = 0$

$x = \dfrac{-(-1) \pm \sqrt{(-1)^2 - 4(1)(1)}}{2(1)} = \dfrac{1 \pm \sqrt{-3}}{2}$

No real Solutions

C.N. $\to \pm 1$

C.P. $\to (\pm 1, f(\pm 1))$

we only need 1 because of $\boxed{[0,3]}$.

$f(1) = \dfrac{1}{1^2 - 1 + 1} = \dfrac{1}{1} = \boxed{1}$

So
$f(0) = 0$ $\quad$ Abs. Min at $(0,0)$

$f(1) = 1$ $\quad$ Abs. Max @ $(1,1)$

$f(3) = \frac{3}{7}$

Apr 8-9:39 AM

$f(x) = 2 + 3x^2 - x^3$

1) find all x-values where $f'(x) = 0$ or undefined.

$f'(x) = 6x - 3x^2$

$6x - 3x^2 = 0$

$3x(2 - x) = 0$

$x = 0$
$x = 2$ $\Rightarrow$ C.N.

2) find all x-values where $f''(x) = 0$ or undefined.

$f''(x) = 6 - 6x$

$6 - 6x = 0$

$x = 1$ $\Rightarrow$ Possible inflection Points @ $x = 1$

Apr 8-9:48 AM